

Legendre Analysis of Hadronic Reactions

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Expansions over Legendre functions are suggested as a model-independent way of compact presentation of modern precise and high-statistics data for two-hadron reactions. Some properties of the expansions are described.

Modern detectors combined with modern accelerator facilities are capable to provide tremendous sets of experimental data. Here the problem arises, how to present those numerous detailed data. For relatively simple cases of $2 \rightarrow 2$ reactions, there are two popular methods. The data are usually presented either as pictures with tens of small panels showing angular distributions for different fixed energies or as similar multi-panel pictures showing excitation functions, *i.e.*, energy distributions at different fixed angles (see *e.g.*, Refs. [1–3]). Both approaches may be used, of course, to check various models, but are not practical for any direct extraction of information. They both are very bulky and non-visual, especially if one would try to present the whole data-set.

As an alternative, we have suggested to expand experimental data into series of the Legendre functions. For unpolarized differential cross sections, it means decomposition

$$\frac{d\sigma}{dz}(E, z) = \sum_{J=0}^{\infty} A_J(E) P_J(z), \quad (1)$$

where E is the c.m. energy, $z = \cos \theta$, and θ is the polar c.m. angle. Formally, this series is infinite. However, in real situations only some finite number of the Legendre coefficients $A_J(E)$ are efficient, since higher coefficients have errors that exceed fitted values of those coefficients. This was illustrated in Refs. [1, 2] for photoproduction reactions $\gamma p \rightarrow \eta p$ and $\gamma p \rightarrow \pi^0 p$ respectively. As a result, the whole set of data appear to be presented in a compact form of energy dependencies for several Legendre coefficients.

Let us briefly discuss properties of the coefficients A_J . Each of them is a sum, formally infinite, of terms bilinear in respect to partial-wave amplitudes. Their angular momenta, j_1 and j_2 , satisfy the familiar conditions

$$|j_1 - j_2| \leq J \leq j_1 + j_2. \quad (2)$$

The summations over j_1 and j_2 should go to infinity. However the partial-wave amplitudes decrease at high j (exponentially in asymptotics), so, when accounting for experimental uncertainties, one needs only a finite number of summands.

The Legendre coefficients have also other, less evident properties. For parity conserving reactions, some of those properties are related to partial-wave amplitudes for states of definite parity. In the unpolarized cross section the positive- and negative-parity amplitudes always appear symmetrically (that is why the unpolarized cross section by itself does not allow to determine the parity of a particular state). It is not quite so for A_J . One can show that the Legendre coefficients provide specific discrimination of parities: A_J with odd J contain only interferences of states with opposite parities, while A_J with even J contain only interferences of states with the same parities, positive or negative. And, of course, only the even- J coefficients may contain squares of absolute values of various partial-wave amplitudes.

It is interesting to discuss how resonances reveal themselves in the Legendre coefficients. Let us consider the simplest idealized case of a pronounced resonance with spin S over very small (negligible) background. Then we practically have only one partial-wave amplitude with $j = S$. According to frequent belief, cross section in such a case contains the resonance contributions into the Legendre harmonics with J up to $2S$. But this is not quite true. As explained above, the resonance contributions (with the same parity and without background!) may appear not in every A_J , only in coefficients with *even* J -values. Moreover, for fermionic resonances having half-integer S , the value of $2S$ is odd, hence A_J with $J = 2S$ does *not* contain the resonance contribution! Of course, a boson resonance having integer S does contribute to A_J with $J = 2S$.

The situation changes if there are background contributions, especially if they may have both positive and negative parities. Due to interferences resonance-background, the resonance signals become present in every A_J , with either even or odd value of J . Moreover, if the resonance is sufficiently intensive, as *e.g.*, $\Delta(1232)$, the resonance interference can work as an amplifier (see examples in Ref. [4]). It can enhance contributions of very small partial-wave amplitudes, which, by themselves, are not seen *vs.* experimental errors. Here we encounter an interesting point. If parity of the resonance is known, parity of interfering states becomes also known, due to the discrimination of parities between even and odd values of J , as explained above.

The Legendre decomposition may be applied also to polarization variables, more exactly, to polarization variables multiplied by the unpolarized cross section. However, instead of Legendre polynomials, one should generally use other functions, associated Legendre functions or even Wigner d -harmonics. Such approach was applied to experimental data on the beam asymmetry [3], which was expanded into a series over the associated functions $P_J^2(z)$.

We can summarize the above statements in the following way:

- Legendre expansions provide a model-independent approach suitable for presentation of modern detailed (high-precision and high-statistics) data for two-hadron reactions;
- This approach is applicable both to cross sections and to polarization variables; it is much more compact than traditional methods, at least, at not very high energies;
- The Legendre coefficients reveal specific correlations and interferences between states

of definite parities;

- Due to interference with resonances, high-momentum Legendre coefficients open a window to study higher partial-wave amplitudes, which are out-of-reach in any other ways.

More detailed discussion of the Legendre expansions, including proofs of various properties stated above as well as examples of physical information which may be obtained by their applications, will be given in a separate paper [5].

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References

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